Econ 802

Second Midterm Exam

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All questions have equal weight. If something is unclear, please ask. In the consumer theory questions you can assume that prices and income are strictly positive.

1. Jane has a preference ordering over consumption bundles $x = (x_1 \, . \, . \, x_n) \geq 0$. Her preferences are complete, reflexive, transitive, and continuous.

(a) Can we be sure that Jane's utility maximization problem has a solution? Provide a detailed justification for your answer.

(b) Define local non-satiation of preferences. What useful thing does this idea imply? Justify your answer. Then define strict quasi-concavity of the utility function, say what useful thing this idea implies, and justify your answer.

(c) State the sufficient second order condition for utility maximization. How is this condition derived? What useful thing does it imply? Justify your answer.

2. Jean has the utility function $u = \min \{a_1x_1, a_2x_2 \, . \, . \, . \, a_nx_n\}$ where $a_i > 0$ for all i. We assume $n \geq 3$ and consider consumption bundles such that $x = (x_1 \, . \, . \, x_n) \geq 0$.

(a) Compute Jean's Hicksian demand functions and her expenditure function. Explain your reasoning.

(b) Compute Jean's Marshallian demand functions and her indirect utility function. Explain your reasoning. <u>Hint</u>: use your results from part (a) and solve for the indirect utility function first.

(c) Define the concept of the inverse demand function. Then discuss whether this concept makes sense for someone with Jean's preferences. Use a graph to explain your reasoning for the case of two goods.

3. Joan has the utility function $u(x_1, x_2) = a \ln x_1 + b \ln x_2$ where $x_1 > 0$, $x_2 > 0$, $a > 0$, and $b > 0$.

(a) Compute Joan's Marshallian demand functions and her indirect utility function.

(b) Suppose the price vector changes from p to q. What new level of income m' does Joan need in order to make her exactly as well off at (q, m') as she was at (p, m)?

First draw a graph with x_1 and x_2 on the axes and carefully explain in words what you are trying to do. Then use math to solve for m' as a function of m, p, and q. Hint: you may want to transform the utility function in a convenient way.

(c) Go back to the original situation (p, m) from part (a). Now suppose the prices change to some new vector q, while at the same time Joan's income changes in such a way that she can exactly afford to buy her previous bundle x(p, m) at the new prices q. At the new prices and income level, is Joan better off, worse off, indifferent, or is the outcome uncertain? Explain using a graph.

4. June's indirect utility function can be written in the form $v(p, m) = w(p)m$. This implies that her expenditure function is $e(p, u) = u/w(p)$.

(a) Compute June's Marshallian and Hicksian demand functions for good i.

(b) Use the Slutsky equation to split the slope of the Marshallian demand $\partial x_i/\partial p_i$ into a substitution effect and an income effect. Notice that we are using the derivative of the quantity of good i with respect to good i's own price. Your expressions for the substitution and income effects should only involve w(p), derivatives of this function, and income (m). They should not include the utility level u explicitly.

(c) Suppose there are many consumers with preferences of this form. Under what conditions would it be possible to aggregate the Marshallian demands of these consumers and treat the market demands as if they came from the maximization of an 'aggregate' utility function? Assuming this is possible, what restrictions (if any) would this impose on the behavior of the market-level demand functions?

5. Here are some miscellaneous questions about market equilibrium.

(a) In the short run there are n identical firms in a market and each firm has fixed cost $F > 0$. How, if at all, would an increase in F for all firms simultaneously affect (i) the output supply function of a typical firm; (ii) the market equilibrium price; and (iii) the profit of a typical firm? Explain.

(b) A typical firm has the long run average cost $a + by^2$ where $a > 0, b > 0$. Describe the firm's output supply function algebraically and show it on a graph. Then use graphs to explain how market equilibrium would be determined in the following two cases: (i) the number of firms n is fixed; (ii) free entry is allowed.

(c) A typical firm has the long run average cost $A + By + Cy^2$ where $A > 0, B < 0$, and $C > 0$ with $4AC > B^2$. Show the LAC and LMC curves on a graph and also indicate the firm's output supply function graphically (you don't need to solve for it algebraically). If the number of firms is fixed at n, can there be a problem with non-existence of market equilibrium? Use graphs to explain your reasoning.